1 Multiple Choice

Question 1	a	b	c	d	e	f
Question 2	a	b	c	d	e	f
Question 3	a	b	c	d	e	f
Question 4	a	b	c	d	e	f
Question 5	a	b	c	d	e	f
Question 6	a	b	c	d	e	f
Question 7	a	b	c	d	e	f
Question 8	a	b	c	d	e	f
Question 9	a	b	c	d	e	f
Question 10	a	b	c	d	e	f
Question 11	a	b	c	d	e	f
Question 12	a	b	c	d	e	f
Question 13	a	b	c	d	e	f
Question 14	a	b	c	d	e	f
Question 15	a	b	c	d	e	f
Question 16	a	b	c	d	e	f
Question 17	a	b	c	d	e	f
Question 18	a	b	c	d	e	f
Question 19	a	b	c	d	e	f
Question 20	a	b	c	d	e	f
Question 21	a	b	c	d	e	f
Question 22	a	b	c	d	e	f
Question 23	a	b	c	d	e	f
Question 24	a	b	c	d	e	f
Question 25	a	b	c	d	e	f

2 Written Response

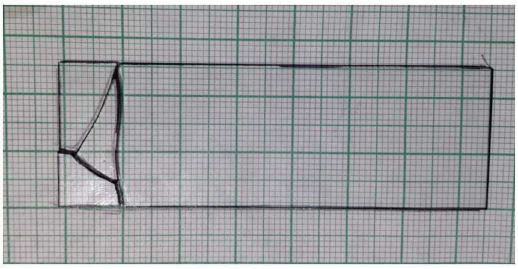
Problem 1

When an objects breaks, many of the molecular bonds get detached, but in this process, some of the energy that caused the break turns into heat and sound wave energy.

In this problem we want to understand what percentage of the energy that causes a break is actually used to break the molecular bondings. As a simple model we can think of glass as a cubical structure, which means each SiO_2 molecule occupies a cube of side length a, and each cubic molecule site has 1 bonding with each of its neighbouring cubes. The energy required to break this bond is called bonding energy, denoted by E_b.

Furthermore, in this problem we are only interested in the **order of magnitude** of the values we obtain.

The following image shows a piece of glass of dimensions 25 mm $\times$75 mm $\times$1 mm that is fallen down from a height 150 cm and broken into pieces.



a) Using the image, estimate the total length of cracks and thereby the total number of broken bonds.

b) Using the latent heat of vaporization for glass, estimate the bonding energy of glass.

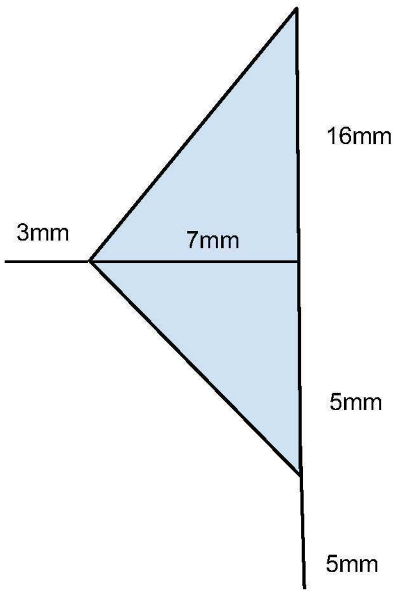
c) What percentage of the energy in this collision is used to break the bonds?

Numerical Values:

- Density of glass: $\rho_g = 2\ \mathrm{g/cm^3}$
- Molar mass of SiO_2: $M_{SiO_2} = 60\ \mathrm{g/mol}$
- Latent heat of Vaporization for glass: $L_g = 10\ \mathrm{kJ/g}$

Solution

a) From the figure, the glass cracked into 4 pieces. The middle piece is approximately triangular and should be divided into two triangles as in the figure below. The hypotenuses of these two triangles are determined from Pythagoras' theorem to be 17.5mm and 8.6mm for the top and bottom sides respectively.



Adding up all the side lengths gives $3 + 17.5 + 8.6 + 16 + 5 + 5 = 55.1$mm. Acceptable answers can differ from this by several millimetres due to the irregularity of the cracks.

Now, the volume occupied by a single molecule is

$$V = \frac{(60\mathrm{g/mol})}{(2\mathrm{g/cm^3})(6.022 \times 10^{23}\mathrm{molecules/mol})} = 4.98 \times 10^{-23}\mathrm{cm^3/molecule}. \quad (1)$$

This is the volume of a cube with side length $a = V^{1/3} = 3.7 \times 10^{-8}cm= 3.7 \times 10^{-7}$mm. Each face of the cube has an area of $1.37 \times 10^{-13}\mathrm{mm}^2$, with one bond protruding from each face.

The glass is 1mm thick, so the total surface area of the cracks is 55.1mm$\times$1mm= 55.1mm^2. Thus the total number of broken bonds is

$$N = \frac{55.1\text{mm}^2}{1.35 \times 10^{-15}\text{mm}^2/\text{bond}} = 4.02 \times 10^{14}. \tag{2}$$

b) The latent heat of Vaporization is the energy density required to vaporize the glass. That is, to break all the bonds of the glass. There are 6 bonds attached to each molecule, but these are shared with other molecules, so 6 bonds per molecule would be over-counting. Consider a cube containing $n \times n \times n$ molecules. Then there are $(n-1) \times (n-1) \times (n-1) \approx n^3$ bonds oriented along each direction. There are 3 directions along which the bonds can lie (x, y, z), so there are a total of $3n^3$ bonds, or 3 bonds per molecule.

Now, the amount of energy per molecule required to vaporize the glass is:

$$(10\text{kJ/g})(60\text{g/mol})\left(\frac{1\text{mol}}{6.022 \times 10^{23}\text{molecules}}\right) = 9.96 \times 10^{-22}\text{kJ/molecule}. \tag{3}$$

Since there are 3 bonds per molecule, we get a bonding energy of

$$(9.96 \times 10^{-22}\text{kJ/molecule})\left(\frac{1\text{molecule}}{3\text{bonds}}\right) = 3.32 \times 10^{-22}\text{kJ/bond} = 3.32 \times 10^{-19}\text{J/bond}. \tag{4}$$

c) The total energy in the system is just the gravitational potential energy of the glass before it is dropped.

$$E_{\text{total}} = \rho V g h = (0.002\text{kg/cm}^3)(1.875\text{cm}^3)(9.81\text{m/s}^2)(1.5\text{m}) = 5.52 \times 10^{-2}\text{J}, \tag{5}$$

where the volume is determined from the dimensions of the glass $V = (2.5\text{cm})(7.5\text{cm})(0.1\text{cm}) = 1.875\text{cm}^3$. Meanwhile, the energy used to break the glass was

$$(3.32 \times 10^{-19}\text{J/bond})(4.04 \times 10^{14}\text{bonds}) = 1.34 \times 10^{-4}\text{J}. \tag{6}$$

So the percentage of the total energy that was used to break the bonds was

$$\frac{1.34 \times 10^{-4}\text{J}}{5.52 \times 10^{-2}\text{J}} \times 100\% = 0.24\%. \tag{7}$$

The rest of the energy is transferred to thermal energy in the glass and floor.

Problem 2

The main span of the Lion Gate Bridge has a length of 473m. On each end there are the expansion joints like the one on the photo below. They allow the span to expand horizontally without warping the steel frame of the span. One day the temperature in Vancouver changed from –4 to +15 degrees between 6 in the morning and 2 in the afternoon.

a) What was the average speed of the "tooth" in one of the expansion joints?

b) At 6 in the morning a piece of tire rubber fell into a crack in front of one of the "fingers", filling it completely. The rubber was 10 cm long and had a cross section of 4 cm^2. What was the force acting on the rubber along its length at 2 PM?

c) What will be the vertical force needed to pull this piece of rubber vertically to remove it?

Clearly state all the assumptions that you made while solving this problem.

Data:
The elastic constant of such a piece of rubber compressed along its length is 28N/m.
The coefficient of friction between the rubber and steel is 0.35.
The linear Expansion coefficient of steel is $13 \times 10^{-6}\ K^{-1}$.
The linear Expansion coefficient of rubber is $77 \times 10^{-6}\ K^{-1}$.

a) In 8 hours the span increases its length by:

$\Delta L = L0\alpha\Delta T = 473\ m \bullet 13 \times 10^{-6}\ K^{-1} \bullet (15-(-4))K = 0.12\ m$

It expands uniformly in both directions so the tooth moves by 0.06 m

The speed is

$0.06/(8 \bullet 3600) = 2 \bullet 10^{-6}\ m/s = 2\ \mu m/s$

b) The expansion of cm of rubber is negligible so we can assume that it was compressed by 0.06m so force acting on the rubber from the sides is

$F = k\Delta x = 28 N/m \bullet 0.06 = 1.6\ N$

c) It acts on both sides of the rubber so total normal force is twice that = 3.2 N

The force of friction is about $3.2 \bullet 0.35 = 1.1$ N

To lift the rubber one has to apply this force plus weight of rubber. We did not specify the density of rubber (which is $1.1 \bullet 10^3\ kg/m^3$) so one could ignore it or say that this density was similar to water and calculate the weight to be about 0.15 N.

So the force needed to pull the rubber was about 1.2 N

Assumptions:

Uniform expansion of the span of the bridge in both directions due only to the expansion of the steel.

Rubber compressed uniformly without "buckling".

Thermal expansion of the rubber was negligible compared to its compression by the tooth.

Solution to Problem 3.

(a) The portion of energy captured by the planet of radius r that is (upon the problem description) a black body and orbiting the star at the distance d from its center is given by

$$\gamma = \frac{\Delta E_p}{\Delta E_s} = \frac{\pi r^2}{4\pi d^2} = \left(\frac{r}{2d}\right)^2 = 4.534 \text{ x } 10^{-10}$$

(b) Since the planet is a black body, the power from the star, absorbed by the planet, is equal to the power, emitted by the planet in all directions. Or

$$(\Delta E_p / \Delta t) = I_p (4\pi r^2) = \gamma (\Delta E_s / \Delta t)$$

where $(\Delta E_s / \Delta t) = 3.85 \text{ x } 10^{26}$ W.
The average temperature T_p on the planet surface can be found from $I_p = \sigma T_p^4$,

$$T_p = \left[\frac{\gamma(\Delta E_s / \Delta t)}{4\pi r^2 \sigma}\right]^{1/4} = \left[\frac{\Delta E_s / \Delta t}{16\pi d^2 \sigma}\right]^{1/4} = 279 \text{ K.} \quad (1)$$

This corresponds to + 6°C.
The real average Earth surface temperature is 288 K = 15°C. If the Earth had no atmosphere, its average surface temperature of 279 K could theoretically be warm enough to produce life and not freeze or burn the living beings in case it were somehow evenly distributed over the surface. However, the Moon surface is known of being much hotter than 100°C on the side facing our Sun and much cooler than 0°C on the dark side. Very like temperature distribution can be expected for any planet without an atmosphere in the Solar system, including Earth. Water as the main solvent boiling on the planet side, exposed to Sun, would kill proteins, and life would be impossible.

(c) The only chance to calculate the average surface temperature of Neptune is given by assumption that Neptune is also a black body without an atmosphere (which is really not so). According to the Stefan-Boltzmann's law and following the formula (1) for a different distance d_N between Sun and Neptune, the average temperature on the surface is given by

$$T_N = \left[\frac{\Delta E_s / \Delta t}{16\pi d_N^2 \sigma}\right]^{1/4} = T_p \sqrt{\frac{d}{d_N}} = 50.6 \text{ K.}$$